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Vibration analysis of fluid-conveying double-walled carbon nanotubes based on nonlocal elastic theory

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Abstract

This paper presents an analytical solution to a coupled vibration problem of fluid-conveying double-walled carbon nanotubes (DWCNTs) and analyzes the influences of nonlocal effect, aspect ratio and van der Waals interaction on the fundamental frequency. According to the analysis, the results show that the vibration frequencies of the first three modes of DWCNTs are lower than those of single-walled carbon nanotubes (SWCNTs). The trend is more obvious when the flow velocity is high. It can also be found that the velocity of vibration-induced flutter instability for DWCNTs is lower than that of SWCNTs. In addition, the frequency of mode 1 of DWCNTs decreases with increasing nonlocal parameter. However, the frequency increases as the aspect ratio and the van der Waals interaction increased, especially at higher flow velocities.

1. Introduction

Recently, the study of fluid-conveying carbon nanotubes (CNTs) has become of great interest to many researchers [1–4]. This is because CNTs can be used as nanopipes for conveying fluids due to its unique hollow nanostructure and superior mechanical characteristics [5–8]. It is well known that experiments to measure the flow characteristics of fluid-conveying CNTs are quite difficult since a CNT is extremely small. To understand the dynamic behaviors of a CNT conveying fluid, molecular-dynamics (MD) simulation is a powerful tool for the theoretical study [9–13]. For instance, Hummer *et al* [9] performed MD simulation to study a nonpolar CNT with a one-dimensionally ordered chain of water molecules. They observed pulse-like transmission of water through the nanotube and found a minute reduction in the attraction between the tube wall and water. Chen *et al* [11] examined the influence of nanotube flexibility on the transport diffusion of CH_4 in (20, 0) and (15, 0) nanotubes. They found that the transport diffusivities were extremely large compared to other known materials when flexibility was taken into account. Jeong *et al* [12] studied the torsional responses of hollow and filled CNTs under a combination of tension and torsion using classical molecular-dynamics simulations.

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In addition, the continuum elasticity theory also has been regarded as an effective method [14–17]. For example, Yoon *et al* [14] studied the influence of internal moving fluid on free vibration and flow-induced flutter instability of cantilever CNTs based on a continuum elastic model. Yan *et al* [17] studied flow-induced instability of double-walled carbon nanotubes based on an elastic shell model and found the critical flow velocities and loss of stability are closely related to the ratio of the length to the outer radius.

When the continuum elasticity theory is applied to the analysis of the nanoscale structures, it is found to be inadequate because of ignoring the small scale effect. In order to study the small scale effect on the dynamic analysis of fluid-conveying CNTs, Lee and Chang [18] applied the nonlocal elasticity concept for the vibration analysis. They analyzed the effects of flow velocity on the vibration frequency and mode shape of the fluid-conveying single-walled carbon nanotube (SWCNT) with nonlocal elastic theory. In addition, the van der Waals interaction between inner and outer nanotubes is dependent of its diameter and has a significant influence on driving the oscillation of nanotube oscillators [19] and applying to nanotube bearings [20]. It is important to understand the effect of van der Waals interaction on the vibration behavior of double-walled carbon nanotubes (DWCNTs).

In this paper, the effects of aspect ratio and van der Waals interaction on the vibration frequency of fluid-conveying

DWCNTs using nonlocal elastic theory is studied analytically. This is a coupled vibration problem due to the van der Waals interaction between the inner and outer tubes. As far as we know, in this paper the coupled vibration equation of fluid-conveying DWCNTs is the first to be derived and the analytical solution to the problem is obtained.

2. Analysis

A schematic diagram of a DWCNT with two fixed ends for conveying fluid, as depicted in figure 1, having inner and outer diameters, d_i and d_o, respectively, is considered as a two-layer hollow cylindrical tube. The coupled governing equation of DWCNTs for conveying fluid by considering the effect of van der Waals interaction between the inner and outer tubes using the theory of nonlocal continuum mechanics can be expressed as [14, 21]

$$EI_1\frac{\partial^4 Y_1}{\partial X^4}+2m_\mathrm{f}v\frac{\partial^2 Y_1}{\partial t\,\partial X}+m_\mathrm{f}v^2\frac{\partial^2 Y_1}{\partial X^2}+\frac{\partial^2}{\partial t^2}\times\left[(m_{c1}+m_\mathrm{f})Y_1-m_{c1}(e_0a)^2\frac{\partial^2 Y_1}{\partial X^2}\right]=c(Y_2-Y_1)\quad(1)$$

$$EI_2\frac{\partial^4 Y_2}{\partial X^4}+\frac{\partial^2}{\partial t^2}\left[m_{c2}Y_2-m_{c2}(e_0a)^2\frac{\partial^2 Y_2}{\partial X^2}\right]=c(Y_1-Y_2)\quad(2)$$

where EI_1 and EI_2 stand for the bending rigidities of the inner and outer tubes, $Y_1(X,t)$ and $Y_2(X,t)$ are the bending deflections of the inner and outer tubes, m_{c1} and m_{c2} are the per unit length mass of the tubes, and v and m_f are the uniform mean velocity and the per unit length mass of the flow fluid in the DWCNT, respectively. e_0a is a nonlocal parameter revealing a nanoscale effect of structure. c is the van der Waals interaction between the two tubes. Then a coupled relationship between the two tubes is established. The coupled equation can be reduced to the same equation in [21] when $m_\mathrm{f}=0$. In addition, the two equations become two decoupled differential equations when $c=0$ is assumed. Then the DWCNT is divided into two isolated tubes and they are at two different resonant frequencies to vibrate. The four terms on the left-hand side of equation (1) are the stiffness force of the DWCNT structure, the gyroscopic and centrifugal forces of the flow fluid, and the inertial force of the structure and flow fluid, respectively.

The corresponding boundary conditions are

$$Y_1(0,t)=Y_1(L,t)=\frac{\partial Y_1(0,t)}{\partial X}=\frac{\partial Y_1(L,t)}{\partial X}=0,\quad(3)$$

$$Y_2(0,t)=Y_2(L,t)=\frac{\partial Y_2(0,t)}{\partial X}=\frac{\partial Y_2(L,t)}{\partial X}=0.\quad(4)$$

The boundary conditions given by equations (3) and (4) correspond to conditions of zero displacement and zero slope at $x=0$ and $x=L$, respectively.

The solution of the differential equation given in equations (1) and (2) can be expressed as

$$Y_1(X,t)=w_1(x)\mathrm{e}^{\mathrm{i}\omega t}\quad(5)$$

$$Y_2(X,t)=w_2(x)\mathrm{e}^{\mathrm{i}\omega t}\quad(6)$$

where ω is the complex circular frequency.

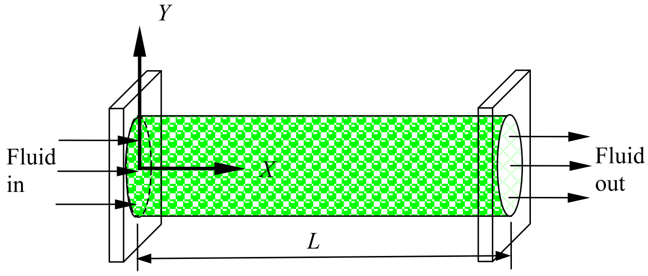

Figure 1. Schematic illustration of a fluid-conveying double-walled carbon nanotube with two fixed ends.
(This figure is in colour only in the electronic version)

The dimensionless variables are defined as

$$x=X/L,\quad y_i=w_i/L,\quad \alpha_i=\frac{I_i}{I},\quad m_i=\frac{m_{ci}}{M},\quad v_n=v\sqrt{\frac{m_\mathrm{f}L^2}{EI}}\quad \beta=\omega\sqrt{\frac{ML^4}{EI}},\quad \delta=\frac{cL^4}{EI},\quad e_n=\frac{e_0a}{L},\quad m=\frac{m_\mathrm{f}}{M},\quad(7)$$

where $M=m_\mathrm{f}+\sum_{i=1}^{2}m_{ci}$, $I=\sum_{i=1}^{2}I_i$ and $i=1,2$ for the inner and outer tubes, respectively.

Using equations (4)–(6), the governing differential equations of motion and the associated boundary conditions can be deduced in the following dimensionless forms:

$$\alpha_1\frac{\mathrm{d}^4y_1}{\mathrm{d}x^4}+2\mathrm{i}\beta v_n\sqrt{m}\frac{\mathrm{d}y_1}{\mathrm{d}x}+v_n^2\frac{\mathrm{d}^2y_1}{\mathrm{d}x^2}-\beta^2\left[(m+m_1)y_1-m_1e_n^2\frac{\mathrm{d}^2y_1}{\mathrm{d}x^2}\right]=\delta(y_2-y_1)\quad(8)$$

$$\alpha_2\frac{\mathrm{d}^4y_2}{\mathrm{d}x^4}-\beta^2\left[m_2y_2-e_n^2m_2\frac{\partial^2y_2}{\partial x^2}\right]=\delta(y_1-y_2)\quad(9)$$

$$y_1(0)=y_1(1)=\frac{\mathrm{d}y_1(0)}{\mathrm{d}x}=\frac{\mathrm{d}y_1(1)}{\mathrm{d}x}=0\quad(10)$$

$$y_2(0)=y_2(1)=\frac{\mathrm{d}y_2(0)}{\mathrm{d}x}=\frac{\mathrm{d}y_2(1)}{\mathrm{d}x}=0.\quad(11)$$

Equations (8)–(11) are a eigenvalue problem for the coupled differential system. The eigenfunctions for the problem can be expressed as

$$y_1(x)=C\mathrm{e}^{\mathrm{i}\gamma x}\quad(12)$$

$$y_2(x)=D\mathrm{e}^{\mathrm{i}\gamma x}\quad(13)$$

where C and D are constants.

Substituting equations (12) and (13) into equations (8) and (9), the following equations are obtained:

$$C[\alpha_1\gamma^4-2\beta\sqrt{m}v\gamma-(v^2+m_1\varepsilon^2\beta^2)\gamma^2+\delta-(m+m_1)\beta^2]-D\delta=0\quad(14)$$

$$-C\delta+D[\alpha_2\gamma^4-m_2\beta^2-m_2\varepsilon^2\gamma^2\beta^2+\delta]=0.\quad(15)$$

Using equations (14) and (15), a polynomial in the variable γ of degree 8 can be obtained and its solution is a function of frequency β. To obtain the frequency equation, equations (12) and (13) can be rewritten as

$$y_1(x) = \sum_{j=1}^{8} C_j e^{i\gamma_j x} \tag{16}$$

$$y_2(x) = \sum_{j=1}^{8} D_j e^{i\gamma_j x} \tag{17}$$

where the complex roots γ_n ($n = 1, 2, \ldots, 8$) are determined from solving equations (14) and (15). Then, substituting equations (16) and (17) into equation (15), we obtain the following relationship:

$$C_n = \lambda_n D_n, \qquad n = 1, 2, \ldots, 8 \tag{18}$$

where

$$\lambda_n = \frac{\alpha_2 \gamma_n^4 - m_2 \beta^2 (1 + \varepsilon^2 \gamma^2) + \delta}{\delta}. \tag{19}$$

Finally, substituting equations (16) and (17) into equations (10)–(11) and using equation (18), the following equation can be yielded:

$$\begin{bmatrix} 1 & 1 & 1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 & 1 \\ \lambda_1 & \lambda_2 & \cdot \;\; \cdot \;\; \cdot & \lambda_8 \\ e^{i\gamma_1} & e^{i\gamma_2} & \cdot \;\; \cdot \;\; \cdot & e^{i\gamma_8} \\ \lambda_1 e^{i\gamma_1} & \lambda_2 e^{i\gamma_2} & \cdot \;\; \cdot \;\; \cdot & \lambda_2 e^{i\gamma_8} \\ \gamma_1 & \gamma_2 & \cdot \;\; \cdot \;\; \cdot & \gamma_8 \\ \lambda_1 \gamma_1 & \lambda_2 \gamma_2 & \cdot \;\; \cdot \;\; \cdot & \lambda_8 \gamma_8 \\ \gamma_1 e^{i\gamma_1} & \gamma_2 e^{i\gamma_2} & \cdot \;\; \cdot \;\; \cdot & \gamma_8 e^{i\gamma_8} \\ \lambda_1 \gamma_1 e^{i\gamma_1} & \lambda_2 \gamma_2 e^{i\gamma_2} & \cdot \;\; \cdot \;\; \cdot & \lambda_8 \gamma_8 e^{i\gamma_8} \end{bmatrix} \times \begin{bmatrix} D_1 \\ D_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ D_8 \end{bmatrix} = 0. \tag{20}$$

The characteristic equation in dimensionless frequencies β can be obtained from solving the above condition. If $\delta \to \infty$ in equation (20) is assumed, the (8×8) matrix can be reduced and written as a (4×4) matrix and then it represents the frequency equation of SWCNTs. The frequency of SWCNTs can be calculated using the (4×4) matrix and the result is the same as obtained from [18].

It should be noted that the equations derived as above can be used to obtain the frequency for the fluid-conveying DWCNT, which is different from the previous studies of DWCNTs [22, 23]. Their method cannot be applied to solve the problem of the fluid-conveying nanotube.

3. Results and discussion

In this paper, the free vibration equation of the fluid-conveying DWCNT has been derived by considering the van der Waals effect and nonlocal elastic theory. The DWCNT can be considered to be composed of two coaxial SWCNTs coupled by the van der Waals interaction. We assume that the two SWCNTs have the same thickness t of 0.35 nm, mass density m_c of 2300 kg m^{-3} and Young's modulus E of 1 TPa [24]. In order to analyze the effect of flow velocity on the frequency of the DWCNT, we assumed that the flowing liquid is water and its mass density is 1000 kg m^{-3}. The DWCNT has an inner diameter d_i of 0.35 nm and an outer diameter d_o of 1.75 nm. According to the calculation, the mass of fluid per unit length is 9.62×10^{-17} kg m^{-2}. EI_1 and EI_2, the bending rigidities for the DWCNT, are 5.9×10^{-26} N m and 4.0×10^{-25} N m, respectively. The dimensionless van der Waals interaction δ is 14 000, which is equivalent to $c = 0.069$ TPa [25]. The other geometric and material parameters used in the calculation are as follows: $e_0 a/L = 0.05$ and $L/d_0 = 10$.

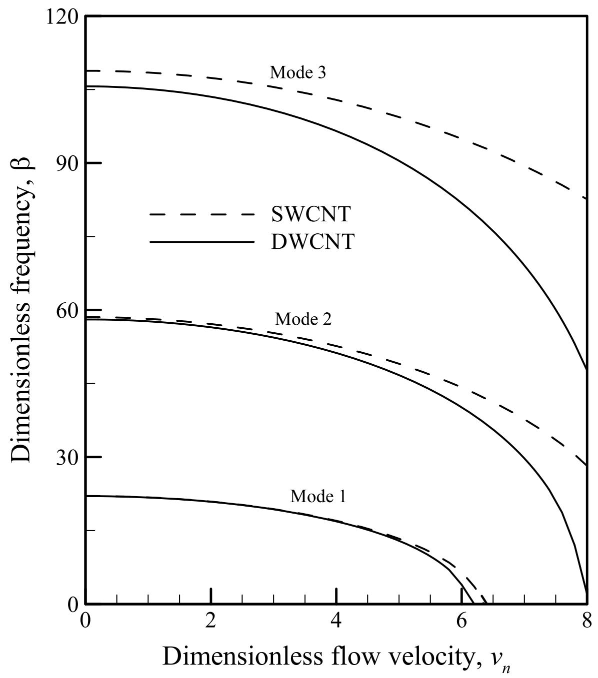

Figure 2. Dimensionless frequencies as a function of dimensionless flow velocity for the first three modes of a clamped–clamped CNT.

In order to compare the results of SWCNTs and DWCNTs, $d_i = 0.35$ nm and $d_o = 1.05$ nm for SWCNTs are assumed in the analysis. Figure 2 shows dimensionless frequencies β as a function of dimensionless flow velocity for the first three modes of the clamped–clamped SWCNTs and DWCNTs. The higher-order vibration modes have the higher frequencies. When $v_n = 0$, the value of β equals the natural frequency of the clamped–clamped CNT. As the flow velocity increases, the value of β decreases. This is because the centrifugal force is proportional to the square of the fluid velocity and it acts in the opposite direction to the stiffness force of the nanotube structure. As the flow velocity increases to about 2π, the value of β is equal to 0 for the first mode. This corresponds to the divergence instability of the CNT. The divergence instability in the second mode takes place when the dimensionless flow velocity is larger than 8. Furthermore, it can be found that the frequency of DWCNTs is lower than that of SWCNTs for the first three modes, especially at high flow velocity. This is because the mass of DWCNTs is larger than that of SWCNTs. The larger mass induces the larger

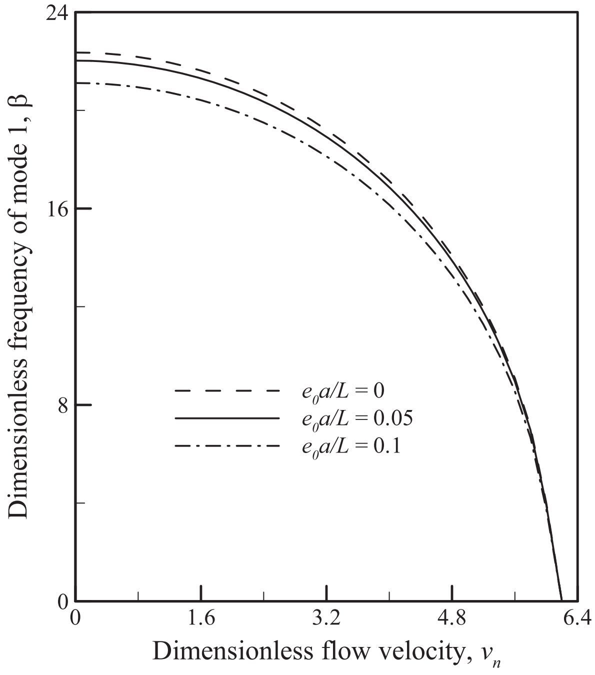

Figure 3. Dimensionless fundamental frequency as a function of dimensionless flow velocity for a clamped–clamped DWCNT with different nonlocal parameters e_0a/L.

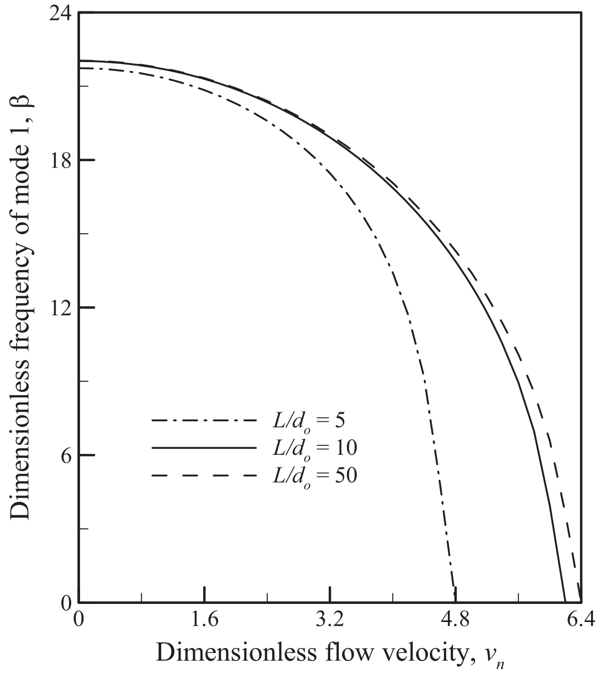

Figure 4. Dimensionless fundamental frequency as a function of dimensionless flow velocity for a clamped–clamped DWCNT with different aspect ratios L/d_o.

centrifugal force at high flow velocity and that make a large decrease in the frequency of DWCNTs. Furthermore, this figure also shows that the velocity of the vibration-induced flutter instability of DWCNTs is lower than that of SWCNTs.

Figure 3 shows the dimensionless frequency β of mode 1 as a function of dimensionless flow velocity v_n for the clamped–clamped DWCNT with different nonlocal parameters e_0a/L. The nonlocal parameter $e_0a/L = 0$ denotes the result obtained using the classical Euler beam model. The frequency of the clamped–clamped DWCNT is significantly influenced by the nonlocal parameter, especially at lower flow velocities. This is because the nonlocal effect due to long range interactions is relatively large at a lower flow velocity. Furthermore, increasing the nonlocal parameter decreases the frequency and the result is similar to that of an SWCNT [18].

The effect of the aspect ratio, L/d, on the fundamental frequency of the clamped–clamped DWCNT is shown in figure 4. With an increase in aspect ratio, the fundamental frequency increases. This implies that the flow velocity of vibration-induced flutter instability increases for a larger aspect ratio. Figure 5 depicts the dimensionless fundamental frequency as a function of dimensionless flow velocity for the clamped–clamped DWCNT with different van der Waals interaction parameters δ. According to equation (7), the relationship between the dimensionless van der Waals interaction parameters δ and length L is given by $\delta = \frac{cL^4}{EI}$. When c, E and I are constant, the value of δ is proportional to L^4. Therefore, the trends are the same in both figures 4 and 5. As the value of δ increases, the frequency of the nanotube increases. This is because the nanotube becomes stiffer due to a more constrained structure.

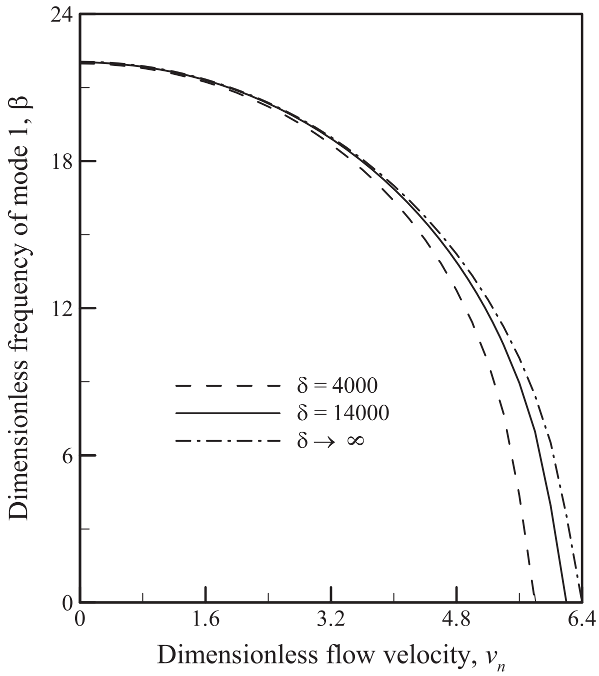

Figure 5. Dimensionless fundamental frequency as a function of dimensionless flow velocity for a clamped–clamped DWCNT with different van der Waals interaction parameters δ.

4. Conclusions

Vibration characteristics of the viscous fluid-conveying DWCNT using continuum elasticity theory were studied, and the effect of geometrical and material parameters on the fundamental frequency were analyzed, including the nonlocal parameter, aspect ratio and van der Waals interaction. The results showed that the frequency of DWCNTs was lower than that of SWCNTs for the first three modes. When the flow velocity was high, this trend was more pronounced. The nonlocal effect on the frequency of mode 1 of DWCNTs became significant when the flow velocity was low, and increasing the nonlocal parameter decreased the frequency. However, the frequency increased as the aspect ratio and the van der Waals interaction increased, especially at higher flow velocities.

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